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ROTATIONAL MAGNETODYNAMICS AND STEERING OF SPACE VEHICLES

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SUMMARY

The stable steering of any vehicle requires that there be applied to it (1) a restoring or directing torque for turning to a required direction, and (2) a damping torque by which all rotations and librations, whether original or caused by the first torque, may be eliminated by immediate dissipation of energy. A damping torque which is predictable and readily available for steering vehicles in outer space is demonstrated by precise studies of the magnetic rotational damping of Vanguard Satellites I and II (1958 Beta 2 and 1959 Alpha 1, respectively).

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INTRODUCTION

Even when artificial satellites were still merely theoretical possibilities, the widely used principle of eddy-current braking had suggested that the spin motion of such freely orbiting bodies would be perturbed and dampened by the various magnetic fields of the earth, of other bodies of the solar system, and of the galactic system. The necessity for predicting, explaining, and analyzing the rotational motions of metallic space vehicles led to the following rigorous derivation of equations of motion for the special cases of rigid axisymmetrical distributions of matter rotating in magnetic fields. Later, when such satellites were actually launched and their rotation had been observed, the expected perturbations and spin decay clearly appeared. Precise numerical confirmation of the formulas was possible for the first two Vanguard satellites, 1958 Beta 2 and 1959 Alpha 1. The surprisingly large damping couple acting on Vanguard II (due to the high magnetic permeability of some of its parts, since the couple varies as the square of the permeability) has suggested the planning and control of such reactions with exterior magnetic fields to stabilize and steer space vehicles.

ELEMENTAL DAMPING TORQUE ON AXIALLY SYMMETRIC CONDUCTORS ROTATING IN A MAGNETIC FIELD

Since this report will derive the magnetic damping of shells and solids of revolution by integration of torques on ring elements, the appropriate couple on an elemental ring

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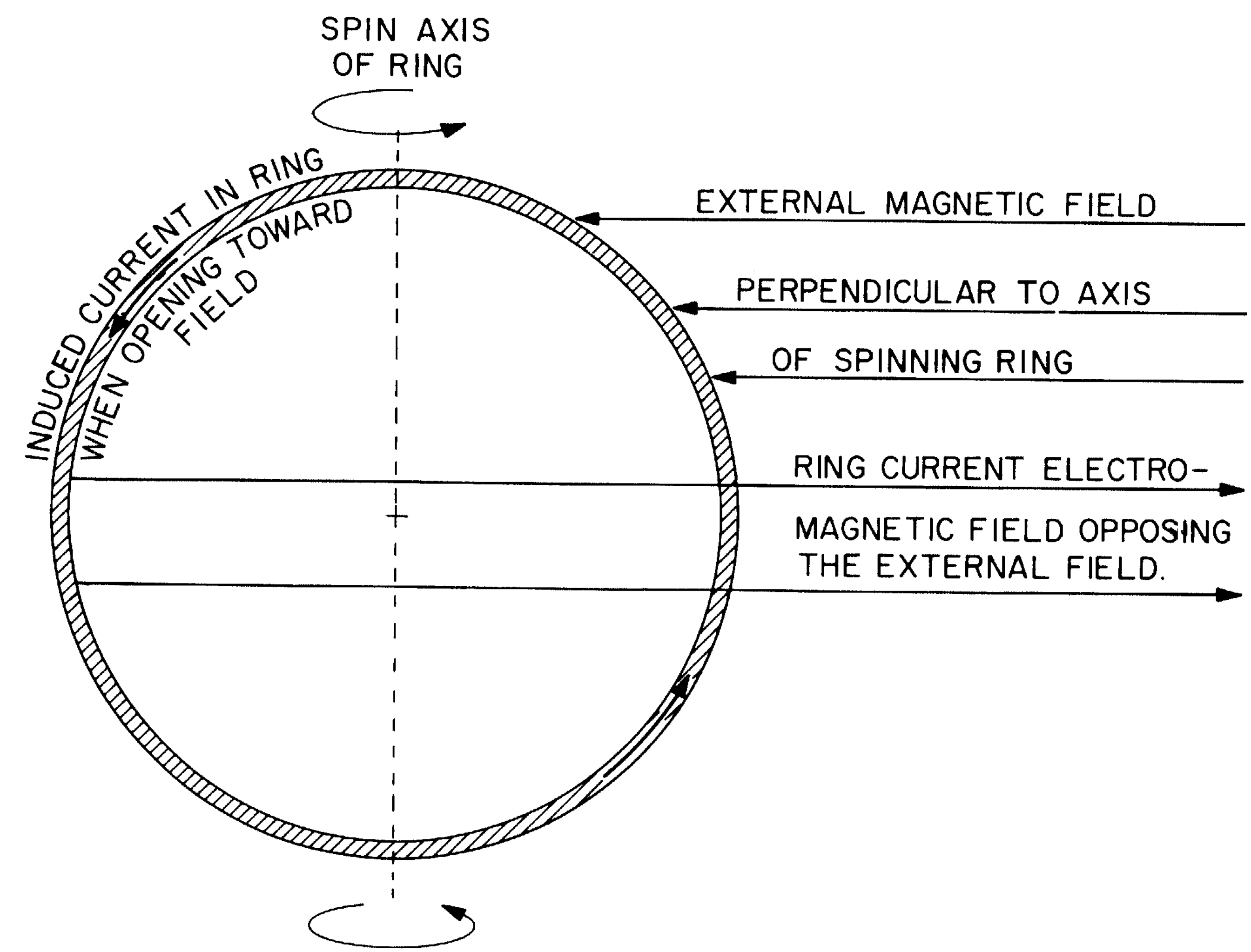

Figure 1 - Interaction of fields on a spinning, conducting ring

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must first be ascertained. Many textbooks (e.g., Reference 1) show demonstrations based on Lenz's Law and other elementary principles that the braking couple C acting at any time t on a thin plane conducting ring of any shape (Figure 1), with enclosed area A and electrical resistance R, rotating with angular velocity ω in a magnetic field with a component H perpendicular to the axis of rotation, is given by

$$C = \frac{A^2\mu^2H^2\omega}{\sqrt{L^2\omega^2 + R^2}} (\cos \alpha \cos^2 \omega t + \sin \alpha \cos \omega t \sin \omega t) , \tag{1}$$

where L is the inductance of the ring, $\alpha = \arctan(L\omega/R)$, μ is the effective magnetic permeability of the ring, and ωt measures the angle from the plane of the ring to the magnetic-field component perpendicular to the spin axis. If this ring is considered an elemental zone of a continuous shell or solid of revolution, the orientation must be considered at all times to be at the maximum value of C (since there will always be a zone of such orientation); whereas, for an insulated ring or solenoid, the *mean* value over a complete cycle should be taken. Only the latter case is considered in the textbooks; therefore the necessary results of the former assumption will be developed here.

Differentiation of Equation 1 with respect to time leads to $\omega t = \alpha/2$ as the condition for maximum C, so that the quantity in parentheses has a maximum value of $(1 + \cos \alpha)/2$. For comparison, the *mean* value (which would be used in the case of an insulated ring) is $(\cos \alpha)/2$, which is less than half as great. By the definitions given with Equation 1,

$$\cos \alpha = \frac{R}{\sqrt{L^2\omega^2 + R^2}} ,$$

and the following equation for the torque on an elemental ring of a continuous shell is obtained:

$$\Delta C = \frac{A^2\mu^2H^2\omega}{2(L^2\omega^2 + R^2)} \left(R + \sqrt{L^2\omega^2 + R^2}\right) , \tag{2}$$

which may be written in the form

$$\Delta C = \frac{A^2\mu^2H^2\omega}{2R} \left[\frac{1 + \sqrt{\left(\frac{L}{R}\right)^2\omega^2 + 1}}{1 + \left(\frac{L}{R}\right)^2\omega^2}\right] . \tag{3}$$

The inductance and resistance of the ring are, respectively,

$$L = b\left(\ln \frac{4b^2}{\pi^2 a} - 4 + \frac{\mu}{2}\right)$$

and (4)

$$R = \frac{b}{\sigma a} ,$$

where b is the length of the ring, σ its conductivity, and a its conducting cross-sectional area. Now, for the elemental ring of a continuous solid, at least one of the dimensions of its cross section a will be infinitesmal, so we may apply L'Hospital's Rule to the ratio given by Equation 4 to find that, as the ring becomes thinner,

$$\lim_{a \to 0} \left(\frac{L}{R}\right) = \lim_{a \to 0} \left(\sigma a\right) = 0, \tag{5}$$

so that terms containing a^2 as a factor will disappear from Equation 3 as infinitesimals of higher order. Thus, substituting Equations 4 and 5 into Equation 3, we obtain

$$\Delta C = \frac{\sigma A^2 \mu^2 H^2 \omega a}{b} \tag{6}$$

as the elemental magnetic couple of a continuous solid.

ROTATIONAL DAMPING OF SPHERICAL SHELLS AND SPHERES

In the case of a spherical body, the elemental ring may be a circular zone perpendicular to, and centered on, a line normal to the spin axis at the center of the sphere (Figure 2). Such a zone is defined by its latitude ϕ, that is, the angle at the center of the sphere from the spin axis to the zone, and by its distance r from the center of the sphere. For such an element, $a = r\,\Delta\phi\,\Delta r$, $b = 2\pi r \cos\phi$, and $A = \pi r^2 \cos^2\phi$, so that the element of couple is, from Equation 6,

$$\Delta C = \frac{1}{2}\sigma\pi\mu^2 H^2 \omega r^4 \cos^3\phi\,\Delta\phi\,\Delta r\,. \tag{7}$$

To obtain the total couple, Equation 7 is integrated throughout the body, giving

$$C = \sigma\pi\mu^2 H^2 \omega \int_{r_0}^{r} r^4 \left[\int_0^{\frac{\pi}{2}} \cos^3\phi\, d\phi\right] dr\,. \tag{8}$$

For a thin spherical shell of thickness Δr, such as the outer structure of an artificial satellite, integration with respect to ϕ alone would give the sufficiently good approximation to the couple:

$$C = \frac{2}{3}\sigma\pi\mu^2 H^2 \omega r^4\,\Delta r\,. \tag{9}$$

Setting this torque in the Newtonian equation of motion as equal to $-I\,d\omega/dt$, where I is the moment of inertia of the body, and integrating, we have

$$t - t_0 = \frac{3I}{2\sigma\pi\mu^2 H^2 r^4\,\Delta r} \ln\frac{\omega_0}{\omega}\,. \tag{10}$$

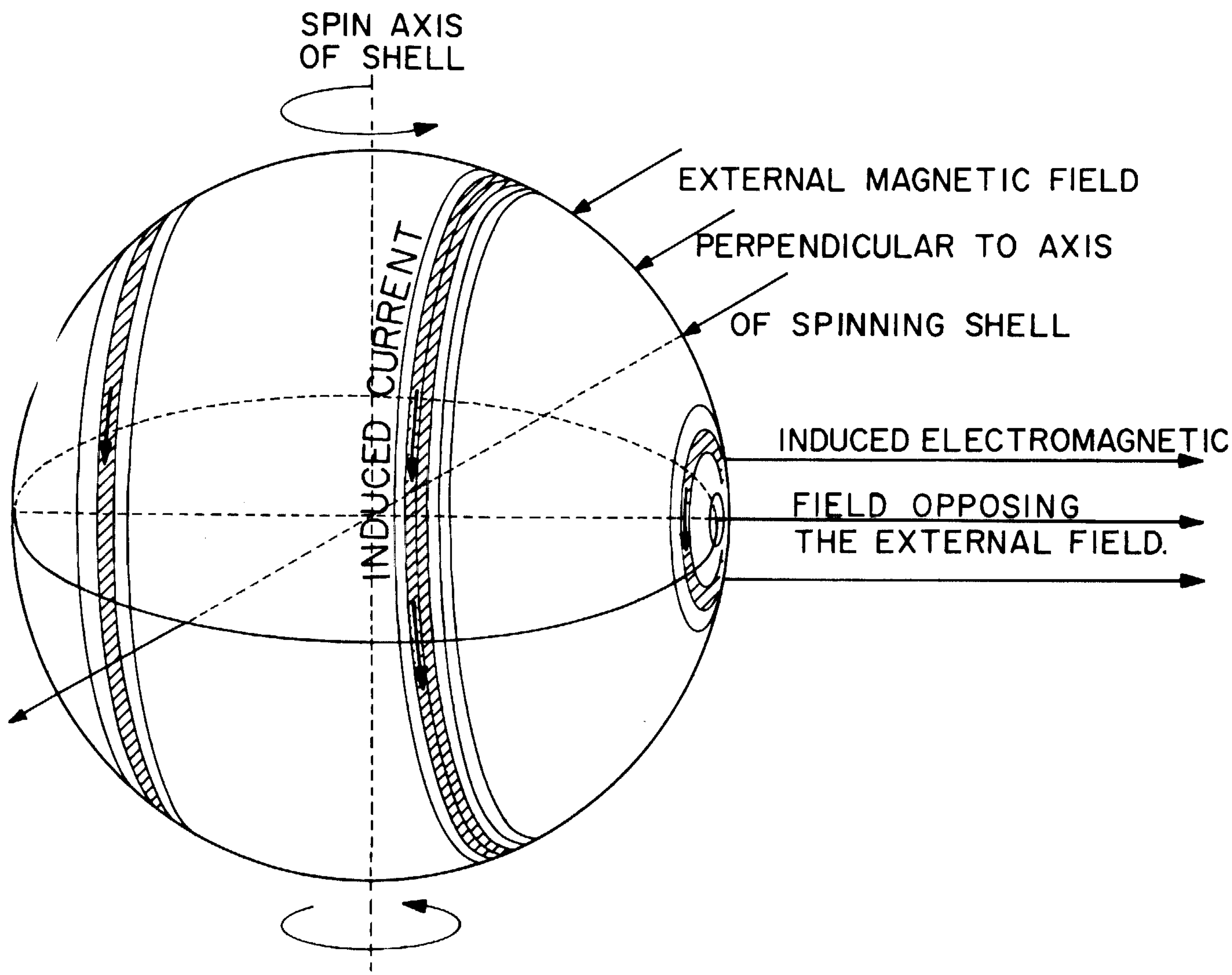

Figure 2 - A spherical shell as an array of rings

For a simple spherical shell of constant density ρ, $I = 8\pi\rho r^4 \Delta r/3$, so that

$$t - t_0 = \frac{4\rho}{\sigma\mu^2 H^2} \ln \frac{\omega_0}{\omega}, \tag{11}$$

which is independent of the dimensions of the shell. Here the coefficient of the logarithm would be the *relaxation time*, i.e., the time required for the angular speed of rotation ω to be reduced by the factor $1/e$.

To obtain the total damping couple on a thick shell or solid sphere, it is necessary to integrate Equation 9 between the limits r_0 and r:

$$C = \frac{2}{15} \sigma\pi\mu^2 H^2 \omega (r^5 - r_0^5) . \tag{12}$$

Setting this torque in the Newtonian equation for rotation as equal to $-I\, d\omega/dt$ and setting $r_0 = 0$, we obtain, for a solid homogeneous sphere,

$$\ln\left(\frac{\omega_0}{\omega}\right) = \frac{2\sigma\pi\mu^2 H^2 r^5}{15\, I} (t - t_0) . \tag{13}$$

But, for a solid homogeneous sphere of radius r and density ρ, $I = 8\pi\rho r^5/15$. Hence,

$$\ln\left(\frac{\omega_0}{\omega}\right) = \frac{\sigma\mu^2 H^2}{4\rho} (t - t_0) , \tag{14}$$

which is independent of the radius and mass of the sphere and is equivalent to Equation 11, as would be expected.

Using a different approach, namely, analysis of heat loss through induced currents as measuring the loss of rotational energy of a solid sphere, Hertz (Reference 2) obtained substantially the same result as Equation 14. However, from a practical standpoint, a relative disadvantage of the Hertz approach would seem to be the difficulty of adapting it to nonspherical conductors or even to spheroids of moderate eccentricity. The extreme simplicity of this result depends on the assumption, implied in the present derivation, that conditions allow effectively instantaneous alignment of induced annular currents, so that the *maximum* element of torque derivable from Equation 1 may be assumed. This assumption tends to weaken under circumstances, not only of high conductivity and magnetic induction (which may be neglected for the present purposes), but especially of large linear rotational velocities of the rotating conductor, as was shown specifically in a later review of the Hertz problem by Gans (Reference 3). However, the inaccuracy of this theory for natural cosmic applications, due to the relatively large dimensions, is partly offset by the relatively lower conductivities, permeabilities, and field strengths usually involved. In any case the inaccuracy is far less than the present numerical uncertainty of these electrical and magnetic properties for astronomical bodies, either natural or artificial.

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ROTATIONAL DAMPING OF NONSPHERICAL BODIES OF REVOLUTION

The damping couple on any continuous solid may be obtained by integrating Equation 6 throughout the body after appropriate expressions for A, the area enclosed by an elemental ring; a, the conducting cross section of this ring; and b, the length of this ring, have been substituted. Numerical integration methods for specific shapes may be used when general analytical methods prove to be difficult or intractable. Schematic formulas for solid prolate and oblate spheroids have been worked out by Gans (Reference 3). Among other solids of revolution having wide application in space vehicles is the cylindrical shell, for which a general solution will now be derived.

Rotation of a Cylindrical Shell About Its Axis of Symmetry

In the case of a right cylindrical shell rotating about its axis of symmetry, the elemental ring would be a rectangle and, for a closed circular cylinder of radius r and length h,

$$\begin{aligned} A &= 2hr\cos\phi \\ a &= \frac{hr\,\Delta\phi\,\Delta r + 2r^2\cos^2\phi\,\Delta\phi\,\Delta r}{h + 2r\cos\phi}, \\ b &= 2(h + 2r\cos\phi), \end{aligned} \tag{15}$$

where ϕ is the central angle in the base of the cylinder from the diameter normal to the magnetic field measured to the chord of the elemental ring. Substituting Equations 15 in Equation 6 leads to the integral for the total couple on a solid cylinder:

$$C = 4\sigma\mu^2H^2\omega h^2 \int_{r_0}^{r} r^3 \left[\int_0^{\frac{\pi}{2}} \frac{(h\cos^2\phi + 2r\,\mathrm{Cos}^4\phi)}{(h + 2r\cos\phi)^2}\,d\phi\right] dr. \tag{16}$$

For a thin cylindrical shell of thickness Δr, integration with respect to ϕ alone yields a sufficiently good approximation to the couple, so that in terms of $k = 2r/h$:

$$C = \sigma\mu^2 H^2 \omega h^4 k \left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2 - 2k}\right) \right. \tag{17}$$

$$\left. \frac{3 - 4K}{k(1-k)\sqrt{1-k^2}} \arctan \frac{\sqrt{1-k^2}}{1 + k} \right] \Delta r,$$

for $k<1$; and

$$C = \sigma\mu^2 H^2 \omega h^4 k \left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2 - 2k}\right) \right. \tag{18}$$

$$\left. - \frac{3 - 4k}{2k(1-k)\sqrt{k^2-1}} \log_e \left(k + \sqrt{k^2-1}\right) \right] \Delta r,$$

for $k>1$. By first setting $h = 2r$ in Equation 16:

$$C = \left(18\pi - \frac{160}{3}\right) \sigma\mu^2 H^2 \omega r^4 \; \Delta r \tag{19}$$

for $k = 1$.

Setting the value of the couple in the Newtonian equation of motion gives, after integration, for any shaped body rotating under a torque C

$$\omega = \omega_0 \; \exp \left[\frac{C(t_0 - t)}{I\omega} \right] . \tag{20}$$

Since, for a thin cylindrical shell rotating about its geometric axis,

$$I = \pi\rho r^3 \; (r + 2h) \; \Delta r, \tag{21}$$

substituting Equations 21 and 17, 18 or 19 into Equation 20 gives the rotational rate ω at time t for such a shell. For the special case $k = 1$, Equation 19 becomes

$$\omega = \omega_0 \; \exp \; \frac{2 \; (27\pi - 80)\sigma\mu^2 H^2 \; (t_0 - t)}{15\pi\rho} , \tag{22}$$

and, like Equation 14 for the sphere, is independent of the mass and dimensions of the cylinder.

Rotation of a Cylinder About an Axis Perpendicular to the Geometrical

For the long cylindrical shells commonly used in space vehicles in which $h > r\sqrt{6}$, rotation about the geometric axis is unstable because the moment of inertia would be greater about a transverse axis perpendicular to the geometrical. In this case, therefore, the existence of any external torque such as magnetic damping would produce a body precession for which the inclination would gradually increase, as in a dying top, until the stable transverse body axis should be attained. Hence, for a more complete discussion of the magnetic damping of a cylinder, it will be necessary to consider rotation about an axis perpendicular to its geometrical axis, although still assuming symmetry of mass and resultant geometrically central location of this transverse axis.

When using the present technique of integrating throughout the body the couple on an elemental conducting ring, two possible forms of this ring must be considered in this case. The first possible element would be a circular ring always perpendicular to and centered on the geometrical axis of the cylinder, and the second a rectangular ring parallel to the geometrical axis such as was used in the preceding discussion of rotation about the geometrical axis.

An important difference from the derivation of damping about the geometrical axis is that in the present specific case there must be used, not the maximum value of each elemental couple, but the mean value, which is half as great. Equation 6, thus modified for each value, would read

$$\Delta C = \frac{\sigma A^2 \mu^2 H^2 \omega a}{2b} . \tag{23}$$

However, the total ΔC would be the sum of those for two such perpendicular elemental rings. To check this principle, consider rotation about the axis of revolution: in this case the corresponding rings are equal, so that the total torque is again the same as Equation 6.

The mean element of the damping couple for the *circular* elemental rings of a cylinder from Equation 23 is

$$\Delta C_1 = \frac{1}{4} \sigma \pi \mu_1^2 H^2 \omega r^3 \, \Delta h \, \Delta r , \tag{24}$$

where r is the dimension perpendicular to the geometrical axis, h the dimension along the axis, and μ_1 the effective mean permeability for a field normal to the axis. The mean element of the couple on the rectangular rings would be half that given by Equation 16:

$$\Delta C_2 = \frac{2\sigma\mu_2 H^2\omega h^2 r^3\left(h\cos^2\phi + 2r\cos^4\phi\right)}{(h+2r\cos\phi)^2}\,\Delta\phi\,\Delta r, \tag{25}$$

where ϕ is the third cylindrical coordinate, and μ_2 should be the mean permeability for a field parallel to the axis of the cylinder. Integrating Equation 25 immediately gives the result for a shell of thickness Δr as half that given by Equations 17, 18 and 19:

$$C_2 = \frac{\sigma\mu_2^2 H^2\omega h^4 k}{2}\left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2-2k}\right) - \frac{3-4k}{k(1-k)\sqrt{1-k^2}}\arctan\frac{\sqrt{1-k^2}}{1+k}\right]\Delta r \tag{26}$$

for $k<1$; and

$$C_2 = \frac{\sigma\mu_2^2 H^2\omega h^4 k}{2}\left[\frac{\pi}{4}\left(\frac{3}{k} + 1 + \frac{k}{2}\right) - \left(1 + \frac{1}{2-2k}\right) - \frac{3-4k}{2k(1-k)\sqrt{k^2-1}}\log_e\left(k + \sqrt{k^2-1}\right)\right]\Delta r \tag{27}$$

for $k > 1$;

$$C_2 = \left(9\pi - \frac{80}{3}\right)\sigma\mu_2^2 H^2\omega r^4\Delta r \tag{28}$$

for $k = 1$.

Integration of Equation 24 for the couple on the circular elements of such a shell should be carried out separately for the ends and for the lateral surface of the cylinder. For the ends, integration is with respect to r, keeping Δh constant as the thickness of the shell. On the other hand, for the lateral surface, integration would be with respect to h, with r and Δr held constant. Thus

$$C_1 = \frac{1}{4} \sigma\pi\mu_1^2 H^2 \omega \left(\frac{1}{2} r^4 \Delta h + r^3 h \Delta r\right) . \tag{29}$$

Since $\Delta h = \Delta r$ for a shell of uniform thickness such as has been assumed in deriving Equations 26 to 28, Equation 29 becomes

$$C_1 = \frac{1}{4} \sigma\pi\mu_1^2 H^2 \omega \left(\frac{1}{2} r + h\right) r^3 \Delta r . \tag{30}$$

Adding the appropriate Equation (26, 27, or 28) to Equation 30 gives the total damping torque, on a cylindrical shell of radius r and height h rotating about a transverse axis, due to a magnetic field H normal to this axis. The corresponding equation of rotational motion is found by substituting the total torque for C in Equation 20, and putting the moment of inertia I equal to that of the cylinder about the axis of rotation. For a shell with ends, rotating about its central transverse axis:

$$I = \frac{1}{6} \pi\rho r \left(3r^3 + h^3 + 6r^2 h + 6rh^2\right) \Delta r \tag{31}$$

In the case of a long, thin tube for which k approaches zero, the relaxation time decreases to zero. This important fact indicates that such tumbling spin about the transverse axis could be made to damp out very quickly by increasing the relative length of the rod. In ferromagnetic materials additional damping as $k \to 0$ would occur as two other effects become prominent, namely, an increase toward the true magnetic permeability of the apparent μ_2 along the geometric axis, and—for certain highly permeable materials which approach saturation in weak fields—an increasing lag in longitudinal magnetization producing hysteresis loss (Reference 4). Since hysteresis damping is linear rather than exponential (Reference 1, p. 285), its relative importance would also rise with decreasing spin-rate.

MAGNETIC DAMPING OF ROTATION OF VANGUARD I (1958 BETA 2)

Since Vanguard I is equipped with a radio powered by solar batteries, its rotation rate, as measured by the signal modulation rate, has now been observed for over 2 years. The resulting data are shown graphically in Figure 3. Regardless of the cause of the apparent rotational damping, these observations may be represented approximately by the empirical equation

$$\omega = 2.72 \exp\left[5.03 \times 10^{-8} (t_0 - t)\right] \text{ rotations per second.} \tag{32}$$

Comparing this equation with Equation 20 and taking $I = 69{,}203$ gm-cm^2, as measured before launching, gives the damping coefficient as $C/\omega = 0.00348$ gm-cm^2/sec.

In order to prove that this observed damping coefficient is essentially magnetodynamical, the theoretical couples for the various satellite parts were computed from Equations 9, 26 or 27, and 30, using the known properties of these parts. A diagram of the satellite is shown in Figure 4, and Table 1 summarizes the details, of which more complete discussions have been published elsewhere (References 5 and 6). Finally, the sum of the theoretical couples of these parts was set equal to the observed damping coefficient, and the equation solved for the effective magnetic field. After proper adjustment for geomagnetic orientation of the spin axis, assuming constancy of this orientation since launching, the resulting mean total field of 0.140 gauss was found to agree with the 0.141 gauss to be expected from ground measurements extrapolated to the mean height of the Vanguard I orbit.

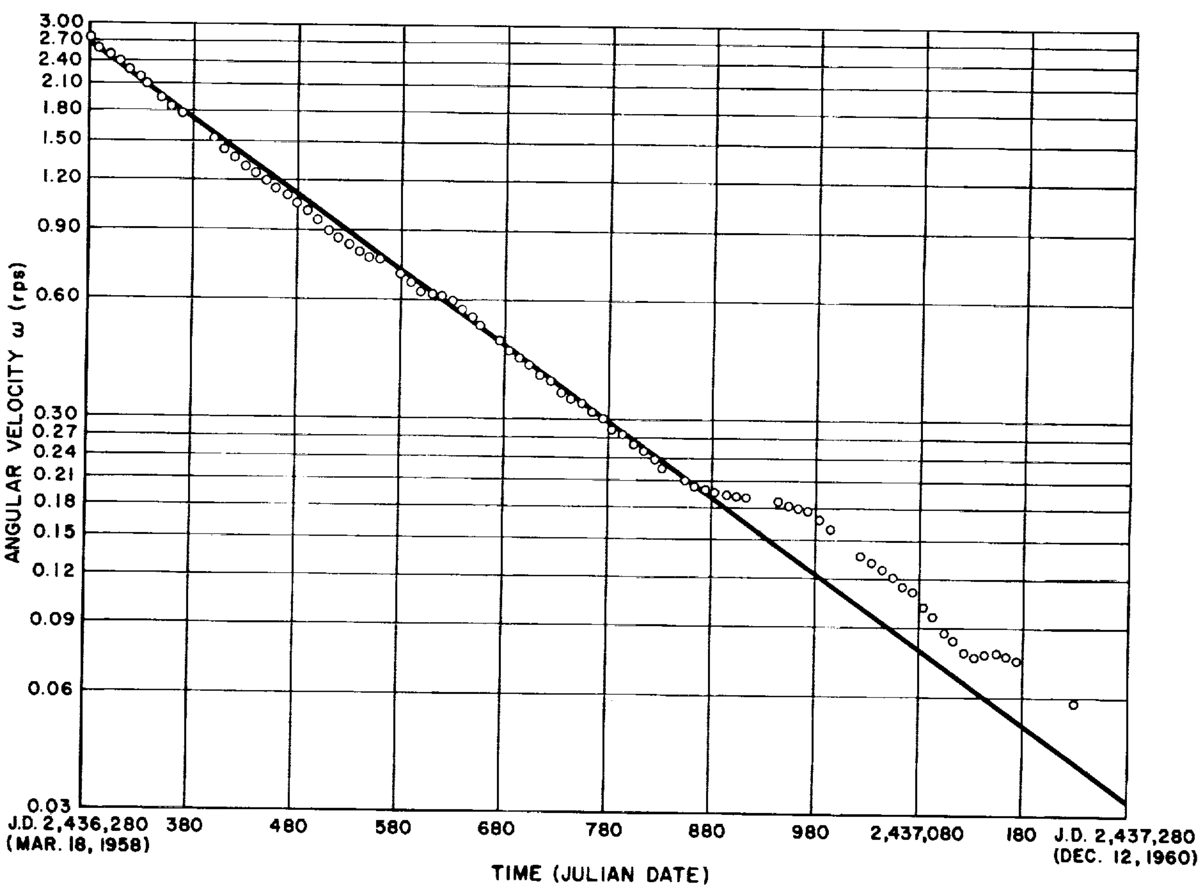

Figure 3 - Observed angular velocity vs time for Vanguard I (1958 Beta 2); straight line represents exponential decay with relaxation time of 230 days

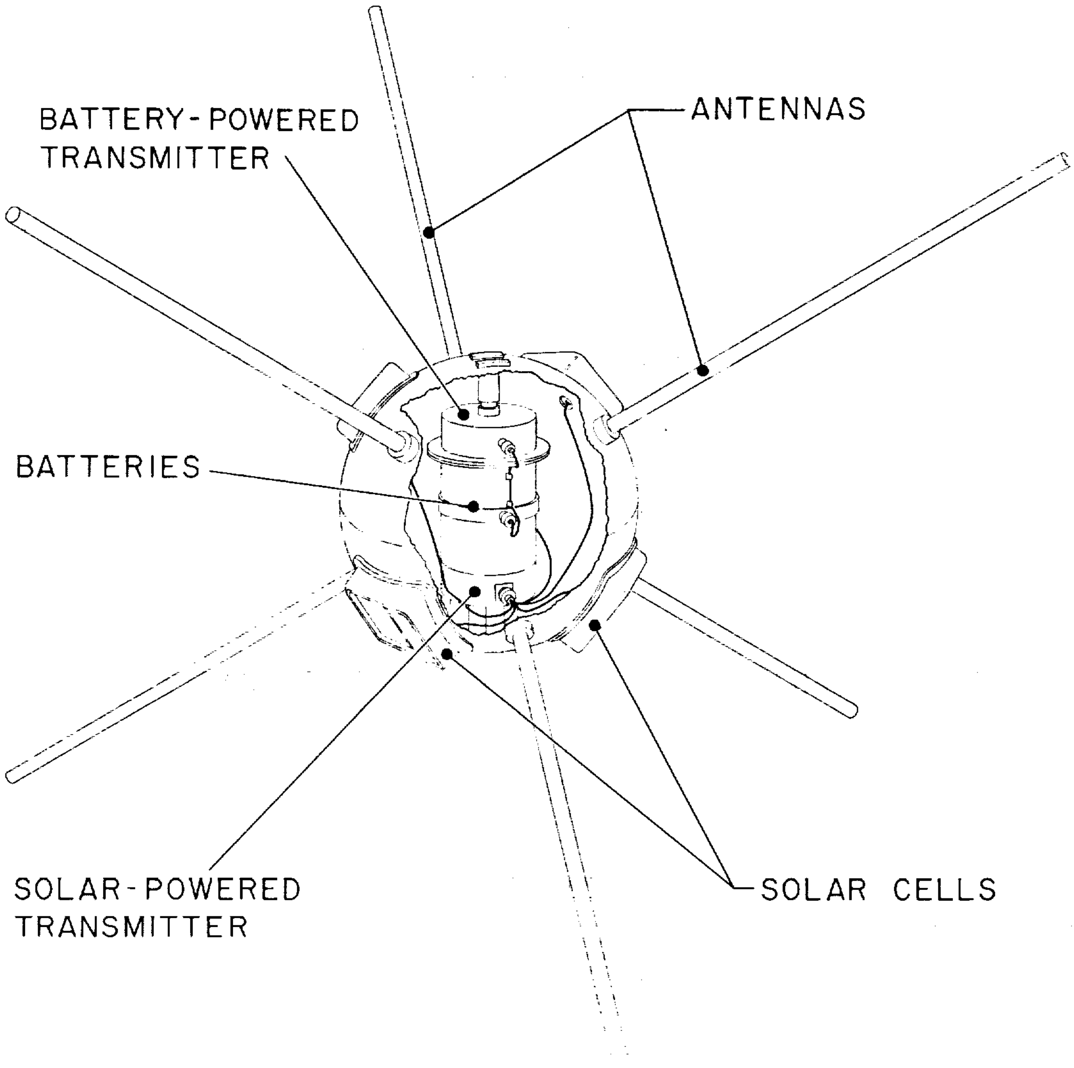

Figure 4 - Cutaway of Vanguard I satellite

With reference to Table 1 it is important to note that, although the couple due to the six antennas was small, the couple due to the less voluminous battery cans accounted for a quarter of the whole; since their permeability was 42 times as great, their resulting couple was nearly 1800 times that for similar nonmagnetic bodies. Also of significance for application to possible magnetodynamic steering mechanisms is the variation of the measured apparent μ of the battery cans, from 65 for a magnetic field parallel to their geometrical axes down to 2 for a field normal to that line.

Table 1

Vanguard I Parts of Magnetodynamic Significance

Part	Shape	r (cm)	h (cm)	Δr (cm)	Material	$1/\sigma$ (emu 45°C)	Effective μ	Equation Used	C/ω $\left(\frac{\text{gm cm}^2}{\text{sec gauss}^2}\right)$
Outer shell	Spherical shell	8.13	--	0.081	Al + Mg 2.5 + Cr 0.25	5×10^3	1	9	$0.151H^2$
Instrument package	Cylindrical shell	2.5	10	0.08	Al	5×10^3	1	26 and 30	$0.005H^2$
6 Antenna cups	Cylindrical shells	2.1	2.2	0.05	Al	5×10^3	1	27 and 30	$0.002H^2$
6 Antennas	Cylindrical shells	0.4	30.5	0.051	Al	5×10^3	1	27 and 30	$0.009H^2$
7 Battery cans	Cylindrical shells	0.79	4.96	0.025	Cold-rolled steel	13.6×10^3	42	26	$0.067H^2$

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MAGNETIC DAMPING OF ROTATION OF VANGUARD II (1959 ALPHA 1)

Radio observations during the four weeks of battery life of Vanguard II yielded the data on its rotation shown in Figure 5. These observations are represented empirically by the equation

$$\omega = 0.25 \exp\left[1.62 \times 10^{-7}\,(t_0 - t)\right] \text{ rotation per second,} \tag{33}$$

for $(t_0 - t)$ in seconds. Again, comparison with Equation 20, with $I = 1.977 \times 10^6$ gm-cm^2 (measured before launching) indicates an observed damping coefficient $C/\omega = 0.3205$ gm-cm^2/sec, whatever its cause. The cause must be essentially magnetodynamic, since equating the sum of the theoretical couples of all parts of the satellite to the observed couple implies an effective field agreeing with the field strength extrapolated from the theory based on surface observations.

A diagram of the Vanguard II satellite is shown in Figure 6. The properties of the satellite parts were more complex in this case than for Vanguard I. In Table 2 only the effective permeabilities and theoretical couples of these parts are given, being sufficient for the present discussion; more complete details have been published elsewhere (Reference 7).

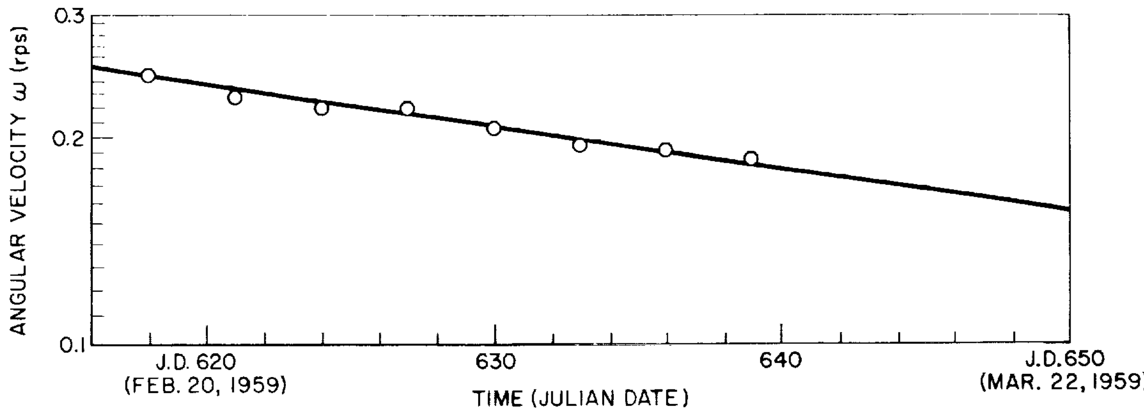

Figure 5 - Observed spin-rate vs time for Vanguard II (1959 Alpha 1); straight line represents exponential decay with relaxation time of 72 days

The sum of the last column of Table 2, set equal to the observed damping couple, is

$$17.148\ H^2 = 0.3205. \tag{34}$$

Solving this gives the effective mean field normal to the spin-axis as 0.137 gauss, which agrees well with the 0.133 gauss to be expected from surface geomagnetic measurements, considering the satellite's axis-orientation and mean orbital height.

However, some of Table 2 should be regarded as a qualitative list of parts having possible magnetodynamic importance rather than as exact numerical data. For the cores of its last three items the permeability was tentatively assumed equal to the true permeability tabulated for such material, although it is likely to be less than this value, due to their small length-to-diameter ratios (Reference 4). On the other hand, no prototype has been tested to determine whether the four permanent magnets with their mu-metal shields may not, indeed, account for a considerable part of the total torque. Properly shaped and standing alone, mu-metal can have an initial permeability as great as 20,000 or more; this would have produced a couple hundreds of times that credited to the SSO-3 transformer cores, thus practically stopping the satellite rotation within a few days. But their shapes, in addition to the fact that these mu-metal shields may be nearly saturated by the permanent magnets they contain, might be expected to reduce the eddy-current torque on them. This suggests possible controllability of the damping couple on a mu-metal shell by retracting it to a position in a saturating field—an engineering application which will be discussed further below.

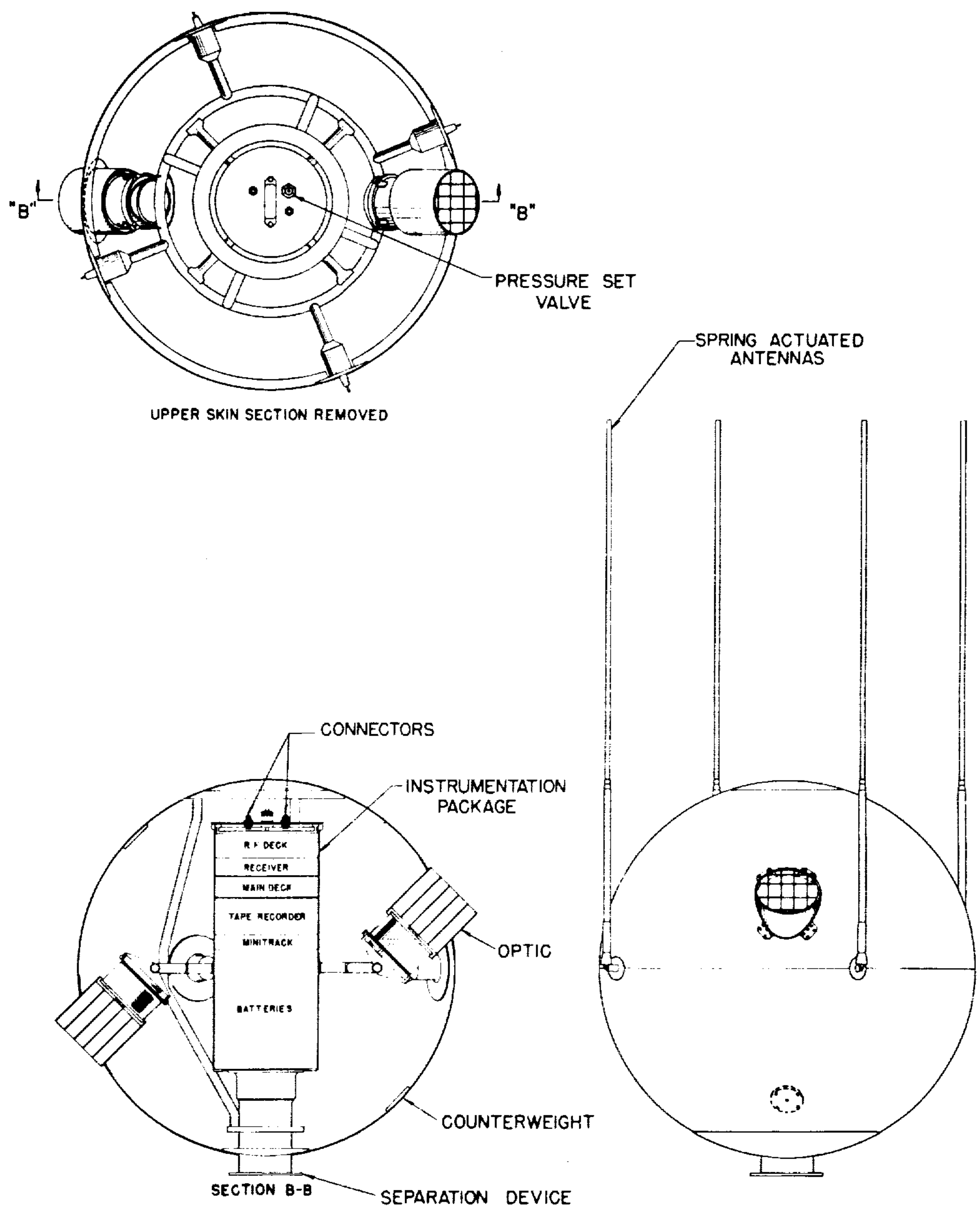

Figure 6 - The Vanguard II satellite (1959 Alpha 1)

Table 2 Vanguard II Magnetodynamics

Part	Effective μ	Equation Used	C/ω $\left(\frac{\text{gm cm}^2}{\text{sec gauss}^2}\right)$
Outer Shell	1	9	$6.364H^2$
Instrument package	1	26 and 30	$0.228H^2$
Structural Tubing	1	26 and 30	$0.004H^2$
Antennas	1	26 and 30	$0.054H^2$
Optical Tubing	< 2	27 and 30	Negligible
4 Permanent Magnets with Mumetal Shields	Neglected	--	Neglected
39 RM-12-R Battery Cans	42	26	$0.371H^2$
23 RM-502-R Battery Cans	48	26	$0.200H^2$
67 RM-640-R Battery Cans	35	27	$0.055H^2$
2 SSO-3 Transformer Cores	2300 (Assumed)	6 (Approximated)	$9.480H^2$
4 DOT Transformer Cores	2300 (Assumed)	6 (Approximated)	$0.369H^2$
5 Relay Cores	500 (Assumed)	26 and 30 (Integrated)	$0.023H^2$

It should be noted that a permanent magnet standing alone suffers mostly a periodic restoring couple given by

$$C = MH \sin \theta , \tag{35}$$

where M is the magnetic moment of the magnet, and θ the angle between its dipole line and the external field vector $\vec{H}$. For the usual magnetically hard material possessing any considerable pole-strength, a relatively small damping couple might be expected in a weak field such as the geomagnetic.

If, in addition to eddy-current damping, there were hysteresis damping, semi-logarithmic plots such as Figures 3 and 5 would be concave downward. Thus, no quantitative evidence of hysteresis damping appears here for either Vanguard I or Vanguard II, although for the latter the extent of the data is too short to be conclusive. In weak fields varied only by slow rotation of the satellite, hysteresis loops—whose area measures this loss—have a considerable width only for special magnetic material (Reference 1, p. 287).

SCIENTIFIC USES OF SPACE VEHICLE MAGNETODYNAMICS

Satellite Vanguard II has, like the planet Venus, no conspicuous surface markings — either radio or optical — by which its rotation could have been tracked after the end of its four-week battery life. However, in lieu of practical arrangements, a simple theoretical extrapolation of Figure 5 indicates that, by October 1960, Vanguard II will be rotating once per orbital revolution, like the earth's natural moon. A further similarity to the moon would be that, since the inclination of the satellite spin axis to the geomagnetic field combined with its revolution around the earth produces an effective geomagnetic rotation, the axis would have drifted to a stable mean orientation roughly parallel to that of the earth. All these effects might be speeded up and accentuated by equipping a satellite with properly devised parts of highly permeable ferromagnetic material, as was indicated in the preceding section, just as aerodynamic drag and radiation pressure effects are accentuated in a balloon satellite.

Indeed, the present magnetodynamic theory may be used as well to discuss the rotational history of the natural astronomical bodies (References 8 and 9). It is thus found to confirm the accepted age of the moon and planets, as well as that of the sun and many other stars with present rotation rates considerably lower than would be expected in the original state of their dynamical systems. For bodies beyond the moon the magnetic field of the galaxy, which has been estimated from cosmic ray research and on other independent grounds to be about 10^{-5} gauss in the present location of the solar system, would always produce some damping couple. Rotation of bodies like Mercury, Venus, and many members of binary systems, located very near to the sun or other stars, may have been slowed by the known magnetic fields of these stars. As yet, there has been no direct measurement of the magnetic fields of other planets, but the 1-spin-per-revolution condition of some of their satellites may be indirect evidence of such fields.

This condition suggests the possible scientific use of space-vehicle magnetodynamics for simple magnetometry, particularly on moon and planetary probes for which long-term radio telemetry might not be feasible or trustworthy. This report has shown how, even without deliberate plans for this purpose, rotational studies of Vanguards I and II have measured the mean geomagnetic field to within a few percent. Proper planning for such measurements would include at least a thorough prelaunch study of the electrical and magnetic properties of all parts of the space vehicle or, best of all, laboratory determinations of a vehicle damping factor describing these properties for the whole vehicle under all expected conditions of rotation, magnetic field, and temperature. It would appear from Equations 20, 9, etc., that the damping time inherent in the vehicle might be measured by

$$K = \frac{I\omega\mu^2 H^2}{C} \text{ sec gauss}^2. \tag{36}$$

To avoid inaccuracies due to mechanical rotation in the laboratory, the procedure might be based on the rotation of only the magnetic vector $\vec{H}$, while measuring the couple C with dynamometers attached to the static vehicle.

Another possible use of vehicle rotational studies would be thermometry. Since the damping couple depends directly on the conductivity of the material, and conductivity is a known (almost linear) function of temperature, the latter quantity might be indirectly deduced even for a radio-dead vehicle from external optical tracking of its rotation (Reference 6).

In a vehicle under some internal control and having highly permeable sleeved tubes whose damping couple is adjustable by varying their length or proportional shielding of permanent magnetic rods, all these scientific uses could have been made much more convenient and accurate. Indeed, such a ferromagnetic rod with adjustable couple could be used as a retractable magnetic "fin" for orienting or steering the vehicle.

APPLICATION OF THE THEORY TO ENGINEERING USES: MAGNETODYNAMICAL STEERING

The steering mechanisms of all earth-bound vehicles, whether on land, water, or in the air, depend on adjustable couple reactions to these respective media. "Steering" is essentially only rotational control of the vehicle and always involves both a damping or accelerating couple by which the rotation may be stopped or started, and a restoring or directing couple which tends to keep the vehicle oriented in the desired direction. The former couple, but not the latter, requires variation of the total rotational energy of the vehicle. Most unaccelerated gyroscopic or gravitational mechanisms for steering a vehicle in the near vacuum of outer space offer only a restoring torque. A bar magnet or solenoid would also offer the restoring couple given by Equation 35.

A damping or accelerating couple is difficult to achieve and to control on a space vehicle. Mechanical expulsion of energy by "yo-yo" devices will dampen rotation, and tangential rockets will either dampen or accelerate it, but both are difficult to arrange for reliable, precise, and stable steering.

The theory and experience with satellite magnetodynamics described herein seems to offer an alternative principle for cosmic steering — especially for achieving a simple and precisely controllable damping torque, since such torque varies as the square of magnetic permeability. Hollow rods of highly permeable material, having their effective magnetic permeability adjusted by variation of their length or magnetic saturation, would provide such a controllable damping torque. They would also provide considerable directional torque in that their apparent permeability is at a maximum when they are parallel with the field (Reference 4).

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Appendix A

List of Symbols

A area projected on plane of angular reference of elemental (thin) closed conducting loop or ring

a conducting cross-sectional area of an elemental loop or ring

b length of elemental conducting loop or ring

C mechanical rotational couple

e base of natural logarithms = 2.718...

H magnetic field component normal to the spin axis

$\vec{H}$ total vector magnetic field

h height of right circular cylinder

I moment of inertia

K inherent vehicle rotational damping factor due to field H

k ratio of diameter to length of a right circular cylinder

L inductance of elemental loop or ring

M magnetic moment of bar magnet; approximately, its length times its pole strength

R resistance of elemental loop or ring

r radius of any body of revolution

t instant of time, or (with ω) elapsed time

α arctan $(L\omega/R)$

θ angle between the external field and the dipole line of a bar magnet or solenoid

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μ effective magnetic permeability

ρ density

σ electrical conductivity, the reciprocal of resistivity

ϕ spherical or cylindrical coordinate of latitude

ω angular velocity

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